Finding Reliable Shortest Paths in Stochastic and Time-dependent Networks for ATIS Applications

Biyu CHEN, William H.K. LAM, Mei Lam TAM & Agachai Sumalee
Outline

- Introduction
- Problem Definition and Model Formulation
- Solution Algorithm
- Numerical Experiments
- Conclusions and Further Study
Conventional routing applications in logistic applications

- Finding shortest distance path
- Without real-time traffic information
Advanced Traveler Information System (ATIS)

Data collection

Data processing

Estimated/predicted Travel time

Data dissemination

The Hong Kong Polytechnic University
RTIS in Hong Kong

Real-time information system (RTIS) in Hong Kong developed by CSE at PolyU and Autotoll Limit. [http://rtis.td.gov.hk/rtis](http://rtis.td.gov.hk/rtis)
Characteristics of link travel times in ATIS

- Discrete (eg. 5 min interval)
- Time dependent (eg. Travel time at 8AM is different from 8PM)
- Stochastic (with travel time prediction error)
Path travel time distribution

- Arrival time at the downstream node depends on the link travel time and arrival time at the upstream node.
Reliable shortest path

Travel Time Budget $\bar{T}_p(t_r)$

Travel Time $X_p(t_r)$

Waiting Time

Expected Travel Time

Safety Margin

Departure Time from Origin $t_r$

Expected Arrival Time $\bar{t}_s$

Actual Arrival Time $t_s$

Preferred Arrival Time $\bar{t}_s$
ATIS-based routing applications

- Multiple routing factors
  - Travel distance
  - Toll charge
  - Travel time
  - Travel time variability
Model formulation

- The objective of RSPP can be formulated as

\[
\min \quad \overline{T_p}(t_r) + (\tau_p + d_p VOD) / VOT
\]

\hspace{1cm} (1)

- Travel time budget satisfy the chance constraint:

\[
\Pr(X_p(t_r) \leq \overline{T_p}(t_r)) \geq \alpha
\]

\hspace{1cm} (2)
Model formulation

\[ \Pr(X_p(t_r) \leq \bar{T}_p(t_r)) \geq \alpha \]

Path travel time  Confidence level

\[ \text{Min } \bar{T}_p(t_r) + (\tau_p + d_p VOD) / VOT \]  \[+\]  \[\bar{T}_p(t_r) \geq \Phi^{-1}_p(\alpha)\]

\[ \text{Min } \Phi^{-1}_p(\alpha) + (\tau_p + d_p VOD) / VOT \]
Model formulation

- The RSPP can be formulated as

\[
\min_{\delta_{ij}} \left[ \Phi_{-1} (\alpha^p) + (\tau_p + d_p V O D) / V O T \right] (3)
\]

S.t.

- Confidence level
- Decision variable
- Value of distance
- Value of time

\[
X_p (t_r) = \sum_{(i, j) \in A} X_{a_{ij}} (t) \delta_{ij} (4)
\]

- Link travel time

\[
\tau_p = \sum_{(i, j) \in A} \tau_{ij} \delta_{ij} (5)
\]

- Link toll charge

\[
d_p = \sum_{(i, j) \in A} d_{ij} \delta_{ij} (6)
\]

- Link length

\[
\sum_{j \in SCS(i)} \delta_{ij} - \sum_{k \in PDS(i)} \delta_{ki} = \begin{cases} 
1 & \forall i = r \\
0, & \forall i \neq r; i \neq s \\
-1 & \forall i = s 
\end{cases} (7)
\]
Challenges for solving RSPP

- Generate path travel time distribution in stochastic and time-dependent network
- Develop efficient algorithm
  - NP-hard problem
  - Large scale network with time-dimension
Generate path travel time distribution

\[ Y_j = Y_i + X_{ij}(t \mid Y_i) \]

1. Discretize

\[ y_{jq_0} = y_{iq_1} + x_{ijq_2}(y_{iq_1}), \quad q_0 = q_1q_2, \quad q_1 = 1,2,...,\eta, \quad q_2 = 1,2,...,\lambda \]

\[ P(Y_j = y_{jq_0}) = \varepsilon \omega, \quad q_0 = 1,2,...,\eta \lambda \]

2. Aggregate

\[ y_{jq} = \left( \sum_{q_0 = (q-1)\lambda + 1}^{q\lambda} y_{jq_0} \right) / \lambda \]

\[ P(Y_j = y_{jk}) = \varepsilon, \quad k = 1,2,...,\eta \]
Exact solution algorithm

- Multi-criterion label correcting algorithms
  - More than one label is maintained at each node
  - NP hard problem

First order dominance
Heuristic solution algorithm

1. Find K reliable shortest path from origin to destination
2. Calculate the path travel time distribution for each kth path
3. Choose the optimal path with minimum utility cost
RTIS in Hong Kong

- Real-time information system (RTIS) in Hong Kong (Tam, M. L. and Lam, W. H. K. 2008)
  - http://rtis.td.gov.hk/rtis
- Extract data from RTIS
  - Choose Wednesdays’ travel time data (4/2008-6/2008)
  - Toll charge from TD

# of nodes = 1379
# of links = 3655
Numerical Example

- Set Richland Gardens (RG) as origin
- Set Central Business District (CBD) in Central as destination
## Reliable shortest paths from RG to CBD

<table>
<thead>
<tr>
<th>Confidence level</th>
<th>VOT (cents / min)</th>
<th>VOD (cents / km)</th>
<th>Path</th>
<th>Arrival Time</th>
<th>Utility cost (min)</th>
<th>Travel cost (HKD)</th>
<th>Expected travel time (min)</th>
<th>Std. dev. of travel time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:29:28</td>
<td>50.61</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.6</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:30:03</td>
<td>51.19</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.7</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:30:40</td>
<td>51.81</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.8</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:31:25</td>
<td>52.56</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.9</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:32:26</td>
<td>53.57</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.99</td>
<td>100</td>
<td>10</td>
<td>Path2</td>
<td>8:34:52</td>
<td>56.01</td>
<td>21.14</td>
<td>29.48</td>
<td>5.35</td>
</tr>
<tr>
<td>0.99</td>
<td>300</td>
<td>10</td>
<td>Path3</td>
<td>8:30:25</td>
<td>39.30</td>
<td>26.64</td>
<td>27.73</td>
<td>1.33</td>
</tr>
<tr>
<td>0.99</td>
<td>500</td>
<td>10</td>
<td>Path3</td>
<td>8:28:26</td>
<td>33.76</td>
<td>26.64</td>
<td>27.73</td>
<td>1.33</td>
</tr>
<tr>
<td>0.99</td>
<td>1000</td>
<td>10</td>
<td>Path1</td>
<td>8:28:26</td>
<td>33.09</td>
<td>46.60</td>
<td>25.89</td>
<td>1.20</td>
</tr>
<tr>
<td>0.99</td>
<td>1000</td>
<td>20</td>
<td>Path1</td>
<td>8:28:26</td>
<td>33.25</td>
<td>48.20</td>
<td>25.89</td>
<td>1.20</td>
</tr>
<tr>
<td>0.99</td>
<td>1000</td>
<td>30</td>
<td>Path1</td>
<td>8:28:26</td>
<td>33.41</td>
<td>49.80</td>
<td>25.89</td>
<td>1.20</td>
</tr>
<tr>
<td>0.99</td>
<td>1000</td>
<td>50</td>
<td>Path1</td>
<td>8:28:26</td>
<td>33.73</td>
<td>53.00</td>
<td>25.89</td>
<td>1.20</td>
</tr>
</tbody>
</table>
Computational performance
Conclusions

- Model for finding reliable shortest path in stochastic and time-dependent networks
- Exact and heuristic solution algorithms
- Comprehensive numerical example
  - Robust to take account travelers’ various preference toward travel cost, travel time and travel time variability
  - Applicable for online routing applications in large-scale road networks
Further studies

- Develop In-vehicle navigation system which should re-optimalize the path en-route
- Develop pick and delivery problem considering travel time uncertainty